Lacunary Fourier series for compact quantum groups

Simeng Wang

Université de Franche-Comté, Besançon
and
Institute of Mathematics, Polish Academy of Sciences, Warsaw

(Great thanks to : Q. Xu, A. Skalski)

Paris, October 2015

Outline

1. Introduction to Fourier series on CQGs
2. Sidon sets and $\Lambda(p)$-sets

Outline

Recall on compact quantum groups

- A compact quantum group is a pair $\mathbb{G} = (A, \Delta)$, where:

  A : a unital C*-algebra, $\Delta : A \to A \otimes A$ a *-homomorphism s.t.

$$(\Delta \otimes \mathrm{id})\Delta = (\mathrm{id} \otimes \Delta)\Delta,$$

$$\overline{\mathrm{span}}((1 \otimes A)\Delta(A)) = \overline{\mathrm{span}}((A \otimes 1)\Delta(A)) = A \otimes A.$$

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- **Classical examples**:
  - $(C(G), \Delta_G)$ with G a compact group, $\Delta_G(f)(s,t) = f(st)$, $f \in C(G)$, $s, t \in G$.
  - $\hat{\Gamma} = (C_r^*(\Gamma), \Delta_{C_r^*(\Gamma)})$ for a discrete group Γ, $\Delta_{C_r^*(\Gamma)}(\lambda(\gamma)) = \lambda(\gamma) \otimes \lambda(\gamma)$, $\gamma \in \Gamma$.

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- There exists a Haar state h on $C(\mathbb{G})$ which is "translate invariant".

Class of unitary representations

- Unitary representation of $\mathbb{G}$: $u = [u_{ij}]_{i,j=1}^{n} \in \mathbb{M}_n(C(\mathbb{G}))$ unitary s.t.

$$\forall 1 \leq j, k \leq n,\ \Delta(u_{jk}) = \sum_{p=1}^{n} u_{jp} \otimes u_{pk}.$$

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- The completions of $\mathrm{Pol}(\mathbb{G})$ wrt different topologies give rise to various typical "function" spaces on $\mathbb{G}$:

$$L^2(\mathbb{G}), \quad C_r(\mathbb{G})(\subset B(L^2(\mathbb{G}))), \quad L^\infty(\mathbb{G})(\subset B(L^2(\mathbb{G}))).$$

  In particular, if $\mathbb{G} = \hat{\Gamma} = (C_r^*(\Gamma), \Delta_{C_r^*(\Gamma)})$, then

$$\mathrm{Irr}(\mathbb{G}) = \Gamma, \mathrm{Pol}(\mathbb{G}) = \mathbb{C}\Gamma, C_r(\mathbb{G}) = C_r^*(\Gamma), L^\infty(\mathbb{G}) = VN(\Gamma).$$

Fourier series

Recall: for a compact group G and $\mu \in M(G)$, $\pi \in \mathrm{Irr}(G)$,

$$\hat{\mu}(\pi) = \int_G u^{(\pi)}(g)^* d\mu(g) \ (\in \mathbb{M}_{n_\pi}(\mathbb{C})),$$

if $d\mu = fdm$ with $f \in C(G)$,

$$\hat{f}(\pi) = \hat{\mu}(\pi) = \int_G u^{(\pi)}(g)^* f(g) dm(g) \ (\in \mathbb{M}_{n_\pi}(\mathbb{C})).$$

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Similarly we define the Fourier series for a CQG $\mathbb{G}$:

For $\varphi \in \mathrm{Pol}(\mathbb{G})'$,

$$\hat{\varphi}(\pi) = (\varphi \otimes \mathrm{id})((u^{(\pi)})^*) = [\varphi(u_{ji}^{(\pi)^*})]_{i,j=1}^{n_\pi} \ (\in \mathbb{M}_{n_\pi}(\mathbb{C})), \quad \pi \in \mathrm{Irr}(\mathbb{G}).$$

For $x \in C(\mathbb{G})$ (or $\mathrm{Pol}(\mathbb{G})$, $L^2(\mathbb{G})$, etc.),

$$\hat{x}(\pi) = \widehat{h(\cdot x)}(\pi) = (h \otimes \mathrm{id})((u^{(\pi)})^*(x \otimes 1)) = [h(u_{ji}^{(\pi)^*} x)]_{i,j=1}^{n_\pi}, \quad \pi \in \mathrm{Irr}(\mathbb{G}).$$

Fourier series

Briefly, we obtain the Fourier transform

$$\mathcal{F} : \mathrm{Pol}(\mathbb{G}) \to c_c(\hat{\mathbb{G}}) := \oplus_{\pi \in \mathrm{Irr}(\mathbb{G})} \mathbb{M}_{n_\pi}(\mathbb{C}), \quad x \mapsto \hat{x}.$$

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The map can be extended to L^p-spaces. Define $\|x\|_1 = \|h(\cdot x)\|_{L^\infty(\mathbb{G})^*}$ for $x \in \mathrm{Pol}(\mathbb{G})$, and let $L^1(\mathbb{G})$ be the completion of $(\mathrm{Pol}(\mathbb{G}), \|\|_1)$. Define $L^p(\mathbb{G})$ to be the complex interpolation space

$$L^p(\mathbb{G}) = (L^\infty(\mathbb{G}), L^1(\mathbb{G}))_{1/p}, \quad 1 \leq p \leq \infty.$$

Define $\ell^p(\hat{\mathbb{G}})$ on $c_c(\hat{\mathbb{G}})$ similarly.

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$$\mathcal{F} : L^p(\mathbb{G}) \to \ell^q(\hat{\mathbb{G}}) \text{ contraction}, \quad 1/p + 1/q = 1.$$

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- (Plancherel formula) For $x \in L^2(\mathbb{G})$,

$$x = \sum_{\pi \in \mathrm{Irr}(\mathbb{G})} d_\pi \mathrm{Tr}(\hat{x}(\pi) F_\pi u^{(\pi)}), \quad \|x\|_2 = \|\hat{x}\|_2.$$

Convolutions

- Recall: For a compact group G and $\mu_1, \mu_2 \in M(G)$,

$$\forall f \in C(G), \quad \int_G f d(\mu_1 \star \mu_2) = \int_{G \times G} f(gh) d\mu_1(g) d\mu_2(h).$$

If $d\mu_1 = f_1 dm, d\mu_2 = f_2 dm$ with $f_1, f_2 \in L^1(G)$, then "define"

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- Similarly, for a CQG $\mathbb{G}$ and $\varphi_1, \varphi_2 \in L^\infty(\mathbb{G})^*$, we may define
$$\varphi_1 \star \varphi_2 = (\varphi_1 \otimes \varphi_2) \circ \Delta.$$
This also induces the convolution $x_1 \star x_2$ for $x_1, x_2 \in L^1(\mathbb{G})$:
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$$h(\cdot x_1 \star x_2) = h(\cdot x_1) \star h(\cdot x_2).$$
- (Liu-W.-Wu, 2015) **Young's inequality**: for $x, y \in \mathrm{Pol}(\mathbb{G})$ we have
$$\|x \star \tau_{\frac{\mathrm{i}}{p'}}(y)\|_r \leq \|x\|_p \|y\|_q, \quad \frac{1}{r} + 1 = \frac{1}{p} + \frac{1}{q}, \frac{1}{p} + \frac{1}{p'} = 1$$
where $1 \leq p, q, r \leq \infty$ if h is tracial, and $1 \leq p, q, r \leq 2$ for general cases (counterexample existing for $r = \infty$). (Rk: similar for LCQGs)

Outline

Sidon sets: classical settings

Definition Let G be a compact abelian group and $\Gamma = \hat{G}$ be the dual group. A subset $E \subset \Gamma$ is called a Sidon set if

$$\forall (\alpha_\gamma) \subset \mathbb{C}, \quad \sum_{\gamma \subset E} |\alpha_\gamma| \sim \| \sum_{\gamma \in E} \alpha_\gamma \gamma \|_{C(G)}.$$

- Various characterizations: interpolation of bounded measures, multipliers, $\Lambda(p)$-estimations, unconditional bases...
- Typical examples: Rademacher functions; lacunarity in $\mathbb{Z}$...

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Noncommutative generalizations:

- $G \rightsquigarrow$ compact non-abelian group, $\Gamma \rightsquigarrow \mathrm{Irr}(G)$;
- $\Gamma \rightsquigarrow$ discrete non-abelian group, $C(G) \rightsquigarrow C(\hat{\Gamma}) := C_r^*(\Gamma)$ group C*-algebra.

Sidon sets: classical settings

Let G be a compact group.
For $f \in L^\infty(G)$ and $\pi \in \mathrm{Irr}(G)$, recall $\hat{f}(\pi) = \int_G f(g) u^{(\pi)}(g)^* dm(g)$.
The ℓ^1-norm on $\hat{f}$ is explicitly given by

$$\|\hat{f}\|_1 = \sum_{\pi \in \mathrm{Irr}(G)} d_\pi \mathrm{Tr}(|\hat{f}(\pi)|).$$

Theorem (Figà-Talamanca) Consider $E \subset \mathrm{Irr}(G)$. TFAE:

1. E is a Sidon set, i.e.,

$$\mathrm{supp}(\hat{f}) \subset E \Rightarrow \|\hat{f}\|_1 \sim \|f\|_\infty;$$

2. $\oplus_{\pi \in E} \mathbb{M}_{n_\pi} = \{\hat{\mu}|_E : \mu \in M(G)\}$;
3. $\oplus^{c_0}_{\pi \in E} \mathbb{M}_{n_\pi} = \{\hat{f}|_E : f \in L^1(\mathbb{G})\}$.

Sidon sets: quantum group setting

Let $\mathbb{G}$ be a compact quantum group.
For $x \in L^\infty(\mathbb{G})$ and $\pi \in \mathrm{Irr}(\mathbb{G})$, recall $\hat{x}(\pi) = (h(\cdot x) \otimes \mathrm{id})(u^{(\pi)})$. The norm on $\ell^1(\hat{\mathbb{G}})$ is explicitly given by

$$\|\hat{x}\|_1 = \sum_{\pi \in \mathrm{Irr}(\mathbb{G})} d_\pi \mathrm{Tr}(|\hat{x}(\pi) F_\pi|).$$

Theorem (W.) Consider $E \subset \mathrm{Irr}(\mathbb{G})$. TFAE:

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Sidon sets: classical settings revisited

If $\mathbb{G} = \hat{\Gamma}$ for a discrete group Γ, recall

$$\mathrm{Irr}(\hat{\Gamma}) = \Gamma \quad \text{and} \quad x = \sum_{\gamma} \hat{x}(\gamma)\lambda(\gamma) \in VN(\Gamma).$$

The previous theorem improves a result of Picardello: (Picardello proved in the special case that Γ is amenable)

Corollary Let Γ be a discrete group (not necessarily amenable). TFAE:

1. $E \subset \Gamma$ is a Sidon set, i.e.,

$$\forall (\alpha_\gamma) \subset \mathbb{C}, \quad \sum_{\gamma \in E} |\alpha_\gamma| \sim \| \sum_{\gamma \in E} \alpha_\gamma \lambda(\gamma) \|_{VN(\Gamma)};$$

2. $E \subset \Gamma$ is a strong Sidon set, i.e.,

$$c_0(E) = \{ f|_E : f \in A(\Gamma)(\cong L^1(\hat{\Gamma}))\}.$$

Various generalizations via multipliers

Multipliers: Each $a = (a_\pi)_{\pi \in \mathrm{Irr}(\mathbb{G})} \in \prod_{\pi \in \mathrm{Irr}(\mathbb{G})} \mathbb{M}_{n_\pi}$ induces a map

$$m_a : \mathrm{Pol}(\mathbb{G}) \to \mathrm{Pol}(\mathbb{G}), \quad m_a x = \mathcal{F}^{-1}(a\hat{x}).$$

We say a is a bounded multiplier on $L^\infty(\mathbb{G})$ if m_a extends to a bdd map on $L^\infty(\mathbb{G})$.

Proposition If $\mathbb{G}$ is coamenable (i.e. $\epsilon : u_{ij}^{(\pi)} \mapsto \delta_{ij}$ is bdd on $C_r(\mathbb{G})$) and $E \subset \mathrm{Irr}(\mathbb{G})$, TFAE:

1. E is a Sidon set;
2. $\oplus_{\pi \in E} \mathbb{M}_{n_\pi} = \{a|_E : a \text{ is a bdd multiplier on } L^\infty(\mathbb{G})\}$;
3. For all $(a_\pi)_{\pi \in E} \in \oplus_{\pi \in E} U(n_\pi)$,

$$\|m_a x\|_\infty \sim \|x\|_\infty, \quad \mathrm{supp}(\hat{x}) \subset E.$$

Sidon sets $\Rightarrow \Lambda(p)$-sets

Definition $E \subset \mathrm{Irr}(\mathbb{G})$ is a $\Lambda(p)$-set if

$$\|x\|_p \sim \|x\|_1, \quad \mathrm{supp}(\hat{x}) \subset E.$$

- Case for compact groups: Hewitt-Ross, Marcus-Pisier;
- Case for (dual of) discrete groups: Picardello, Harcharras...
- Case for compact quantum groups: more delicate –

  Theorem (Blendek-Michališek 13') Let $\mathbb{G}$ be a CQG s.t. the Haar state h is tracial on $C(\mathbb{G})$. Denote

  $$\chi_\pi = \sum_{i=1}^{n_\pi} u_{ii}^{(\pi)}, \quad \pi \in \mathrm{Irr}(\mathbb{G}).$$

  If $E \subset \mathrm{Irr}(\mathbb{G})$ is a Helgason-Sidon set ($\subsetneq$ Sidon set), then for all $(c_\pi)_{\pi \in E} \subset \mathbb{C}$ and $x = \sum_{\pi \in E} c_\pi \chi_\pi$,

  $$\|x\|_2 \sim \|x\|_1.$$

Sidon sets $\Rightarrow \Lambda(p)$-sets

Theorem (W.) Let $\mathbb{G}$ be a CQG. If $E \subset \mathrm{Irr}(\mathbb{G})$ is a Sidon set, then

$$(*) \qquad \|x\|_p \sim \|x\|_1, \quad \mathrm{supp}(\hat{x}) \subset E, \quad 1 \leq p < \infty.$$

Remark: The condition on Sidon set can be weakened:
for ex, weak Sidon set (including free generators), interpolation set of bdd multipliers, etc.

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Unsatisfactory point: The equivalent constant obtained in $(*)$ is $K_p = O(p)$, which is worse than that in the classical case $K_p = O(\sqrt{p})$.

Existence of $\Lambda(p)$-sets

Theorem (Bożejko; W.) Let (M, φ) be a vNa equipped with a normal faithful state φ. Let $B = \{x_i \in M : i \geq 1\}$ be an orthonormal system wrt φ s.t. $\sup_i \|x_i\|_\infty < \infty$. Then for each $1 \leq p < \infty$, there exists an infinite subset $\{x_{i_k} : k \geq 1\} \subset B$

$$\forall (c_k) \subset \mathbb{C}, \quad \|\sum_{k\geq 1} c_k x_{i_k}\|_{L^p(M)} \sim \Big(\sum_{k\geq 1} |c_k|^2\Big)^{\frac{1}{2}}.$$

Corollary Let $\mathbb{G}$ be a CQG. Let $E \subset \mathrm{Irr}(\mathbb{G})$ be an infinite subset with $\sup_{\pi\in E} d_\pi < \infty$. Then for each $1 \leq p < \infty$, there exists an infinite subset $F \subset E$ s.t.

$$\|x\|_p \sim \|x\|_1, \quad \mathrm{supp}(\hat{x}) \subset F.$$

Further remarks: Examples & Questions

We have some examples (Note: not complete analogues of classical objects):

- "generators" for $\prod_{k\geq 1} \mathrm{SU}_{q_k}(2)$, $0<q \leq q_k \leq 1$.
- "generators" for $\prod_{N\geq 1} O_N^+$.
- some explicit central $\Lambda(4)$-set in $\mathrm{SU}_q(2)$ with $q \neq 1$, which does not exist for $\mathrm{SU}(2)$.

Some questions:

- Does the $\Lambda(p)$-sets depend on the choice of interpolation parameter of noncommutative L^p?
- The existence of Sidon sets?
- Is the union of two Sidon sets again a Sidon set?
- The order of $\Lambda(p)$-constant for a Sidon set?
- ...

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[3] Blendek T. and Michaliček J., L^1-norm estimates of character sums defined by a Sidon set in the dual of a compact Kac algebra. *J. Operator Theory*, 70(2):375–399, 2013.

Thank you very much!